Statistics

Fall 2021

Lecture 12

Intro. to odds: (SG 12)

odds are the ratio of two numbers writtin in simplest form using : Notation.

odds in favor of event E are

\# E happens : # $\overline{E}$ happens

Ex: There are 40 people in a meeting. 15 Males & 25 Females

P(Select one Female) = $\frac{25}{40} = \boxed{\frac{5}{8}}$

odds in favor of selecting one female

\# Females : # $\overline{\text{Females}}$

25 : 15

25 [÷] 15 [MATH] [1:] [Enter] → $\boxed{5:3}$

Ex:
I toss a Coin 200 times, I landed 125 tails.

$P(\text{Land tails}) = \dfrac{125}{200} = \boxed{\dfrac{5}{8}} = \boxed{.625}$

125 $\boxed{\div}$ 200 $\boxed{\text{Math}}$ $\boxed{1:}$ $\boxed{\text{Enter}}$ $\boxed{\text{MATH}}$ $\boxed{2:}$ $\boxed{\text{Enter}}$

odds in favor of landing tails.

$\# \text{Tails} : \# \overline{\text{Tails}}$

$125 : 75 \Rightarrow \boxed{5:3}$

125 $\boxed{:}$ 75 $\boxed{\text{Math}}$ $\boxed{1:}$ $\boxed{\text{Enter}}$

odds against landing tails

$\boxed{3:5}$

A standard deck of playing Cards has 52 Cards, 26 Black and 4 Aces.

Find odds in favor of drawing

1) Black Card

$\# \text{Black Card} : \# \overline{\text{Black Card}}$

$26 : 26 \Rightarrow \boxed{1:1}$

2) Ace card

$\# \text{Ace} : \# \overline{\text{Ace}}$

$4 : 48$

$\boxed{1:12}$

3) Black Ace Card.

$\# \text{Black Ace} : \# \overline{\text{Black Ace}}$

$2 : 50 \Rightarrow \boxed{1:25}$

$$\boxed{\begin{array}{l} \text{odds in favor of event } E : a:b \\ \text{odds against event } E : b:a \end{array}}$$

How to $\boxed{\text{find } P(E)}$ when we have $\boxed{\text{odds}}$

If odds in favor of event E are $\boxed{a:b}$,

then $\boxed{P(E) = \frac{a}{a+b}}$ & $\boxed{P(\bar{E}) = \frac{b}{a+b}}$

Ex: Suppose the odds in favor of event E are 3:17,

1) odds against event E are 17:3.

2) $P(E) = \frac{3}{3+17} = \boxed{\frac{3}{20}} = \boxed{.15}$

3) $P(\bar{E}) = \frac{17}{3+17} = \boxed{\frac{17}{20}} = \boxed{.85}$

How to find the odds in favor of event E when P(E) is given:

odds in favor of event E are

$P(E) : P(\bar{E})$ Always reduce

Ex: Suppose $P(E) = .24$

1) $P(\bar{E}) = 1 - P(E) = 1 - .24 = \boxed{.76}$

2) odds in favor of event E.

$P(E) : P(\bar{E}) \Rightarrow .24 : .76 \Rightarrow \boxed{6:19}$

$.24 \boxed{\div} .76$ $\boxed{\text{MATH}}$ $\boxed{1:}$ $\boxed{\text{Enter}}$ $\frac{6}{19}$

3) odds against event E. 19:6

Prob. of passing certain exam on first attempt is 12.5%.

1) $P(\text{passing}) = 12.5\% = \boxed{.125} = \frac{1}{8}$

.125 MATH 1: Enter

2) $P(\overline{\text{Passing}}) = 1 - 12.5\% = 87.5\%$

$= 1 - .125 = .875$

$= 1 - \frac{1}{8} = \frac{7}{8}$

3) odds in passing in first attempt

$P(\text{Passing}) : P(\overline{\text{Passing}}) \rightarrow \boxed{1:7}$

$.125 : .875$

out of 8 students

4) odds against passing in first attempt $\boxed{7:1}$

1 pass

7 $\overline{\text{pass}}$.

odds in the form of bets:

odds in favor of event E are

$a : b$

\$ bet

\$ Net return

Suppose the odds for a game are 3:7.

$3 : 7$

\$3 bet

\$7 Net return

How much do we bet if we wish to make \$350 Net?

$\frac{\$3 \text{ bet}}{\$7 \text{ Net}} = \frac{\$x \text{ bet}}{\$350 \text{ Net}}$

$\frac{3}{7} = \frac{x}{350}$

$7x = 3(350)$

$x = \frac{3(350)}{7}$

$x = 150$

We need to place \$150 bet to make \$350 Net.

$P(A) = .72$ $\quad P(B) = .18$

$P(A \text{ and } B) = .1$

1) $P(\bar{A}) = 1 - P(A) = \boxed{.28}$

2) $P(\bar{B}) = 1 - P(B) = \boxed{.82}$

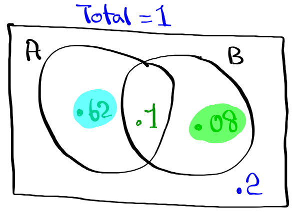
3) $P(A \text{ or } B) =$

$P(A) + P(B) - P(A \text{ and } B) =$

$.72 + .18 - .1 = \boxed{.8}$

4) Venn Diagram

Total = 1



$P(A \text{ only}) = .72 - .1 = .62$

$P(B \text{ only}) = .18 - .1 = .08$

5) $P(A \text{ only or } B \text{ only, not both}) = .62 + .08 = \boxed{.7}$

6) $P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = 1 - .8 = \boxed{.2}$

De Morgan's Law

7) $P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - .1$

$= \boxed{.9}$

Ex:

A and B are $\underbrace{\text{Mutually Exclusive events}}_{\text{Disjoint events}}$.

$P(A \text{ and } B) = 0$

$\boxed{\text{Do not use } \emptyset \text{ for } 0.}$

$P(A) = .65$

$P(B) = .25$

1) $P(\bar{A}) = 1 - .65 = \boxed{.35}$

2) $P(\bar{B}) = 1 - .25 = \boxed{.75}$

3) $P(A \text{ and } B) = \boxed{0}$

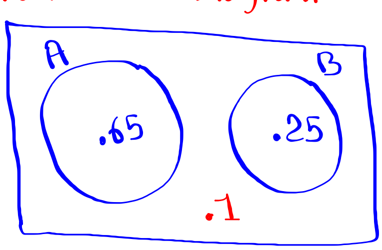
4) $P(A \text{ or } B)$

$= P(A) + P(B) - P(A \text{ and } B)$

$= .65 + .25 - 0 = \boxed{.9}$

5) Construct Venn Diagram

Total=1



Ex:

$P(A) = 95\%$

.95 MATH

1:

Enter

1) P(A) in reduced fraction

$95\% = .95 = \boxed{\frac{19}{20}}$

2) $P(\bar{A})$ in decimal

$1 - P(A) = 1 - .95 = \boxed{.05}$

3) odds in favor of event A.

$P(A) : P(\bar{A}) \Rightarrow .95 : .05 \Rightarrow \boxed{19:1}$

4) odds against event A.

$\boxed{1:19}$

Multiplication Rule:

Keyword AND

Multiple Action event

ex: flip a coin twice

TT TH HT HH

ex: Draw 3 cards from a standard deck of cards

AAA $AA\bar{A}$ $A\bar{A}A$ .... $\bar{A}\bar{A}A$

Case I: Independent events

one outcome does not change the prob. of other outcomes.

$P(\text{Boy}) = .5$ $P(\text{Girl}) = .5$

If A and B are independent events,

$P(A \text{ and } B) = P(A) \cdot P(B)$

$P(B) = .5 \qquad P(G) = .5$

Consider a family with 2 kids

$\underbrace{BB \quad BG \quad GB \quad GG}_{\text{Sample Space}}$

List of all possible outcomes

$P(2\text{ Boys}) = P(B) \cdot P(B) = (.5)(.5) = \boxed{.25}$

$P(2\text{ Girls}) = P(G) \cdot P(G) = (.5)(.5) = \boxed{.25}$

$P(1B \text{ \& } 1G) = P(BG \text{ or } GB) = (.5)(.5) + (.5)(.5) = \boxed{.50}$

# Boys	P(# Boys)
2	.25
1	.50
0	.25

Ex:
A standard deck of playing cards has 52 cards with 12 face cards. (40 $\overline{\text{face}}$)

Draw 2 cards with replacement.

$\underbrace{FF \quad F\bar{F} \quad \bar{F}F \quad \bar{F}\bar{F}}_{\text{Sample Space}}$

$P(FF) = \frac{12}{52} \cdot \frac{12}{52} = \boxed{\frac{9}{169}}$

$P(\bar{F}\bar{F}) = \frac{40}{52} \cdot \frac{40}{52} = \boxed{\frac{100}{169}}$

$P(1F \text{ \& } 1\bar{F}) = P(F\bar{F} \text{ or } \bar{F}F) = \frac{12}{52} \cdot \frac{40}{52} + \frac{40}{52} \cdot \frac{12}{52} = \boxed{\frac{60}{169}}$

# Face	P(# Face)
2	9/169
1	60/169
0	100/169

Ex: A box Contains 2 Dimes and 3 Nickels. Draw 2 Coins with replacement.

Sample Space NN ND DN DD

Total values(¢) 10¢ 15¢ 15¢ 20¢

$P(10¢) = P(NN) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} = \boxed{.36}$

$P(15¢) = P(ND \text{ or } DN) = \frac{3}{5} \cdot \frac{2}{5} + \frac{2}{5} \cdot \frac{3}{5} = \boxed{.48}$

$P(20¢) = P(DD) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = \boxed{.16}$

Values	P(Values)
10¢	.36
15¢	.48
20¢	.16

Ex:

$P(A) = .8$

$P(B) = .5$

A and B are independent events.

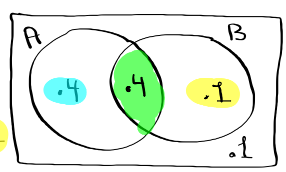
$P(\bar{A}) = 1 - P(A) = \boxed{.2}$ $P(\bar{B}) = 1 - P(B) = \boxed{.5}$

$P(A \text{ and } B) = P(A) \cdot P(B) = (.8)(.5) = \boxed{.4}$

Venn Diagram

$P(A \text{ only}) = .8 - .4 = .4$

$P(B \text{ only}) = .5 - .4 = .1$



$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .8 + .5 - .4 = \boxed{.9}$

Use DeMorgan's Law to Find

$P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - .9 = \boxed{.1}$

$P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - .4 = \boxed{.6}$

Ex:

$P(A) = .6$

$P(B) = .3$

A and B are independent Events.

$P(\bar{A}) = 1 - P(A) = \boxed{.4}$

$P(\bar{B}) = 1 - P(B) = \boxed{.7}$

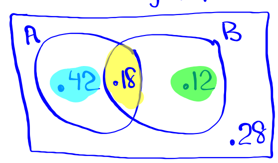
$P(A \text{ and } B) = P(A) \cdot P(B) = \boxed{.18}$

$P(A \text{ or } B) =$

$P(A) + P(B) - P(A \text{ and } B) =$

$.6 + .3 - .18 = \boxed{.72}$

Venn Diagram



$P(A \text{ only OR } B \text{ only}) = .42 + .12 = \boxed{.54}$

$P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = 1 - .72 = \boxed{.28}$

De Morgan's Law

$P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - .18 = \boxed{.82}$

Tree Diagram

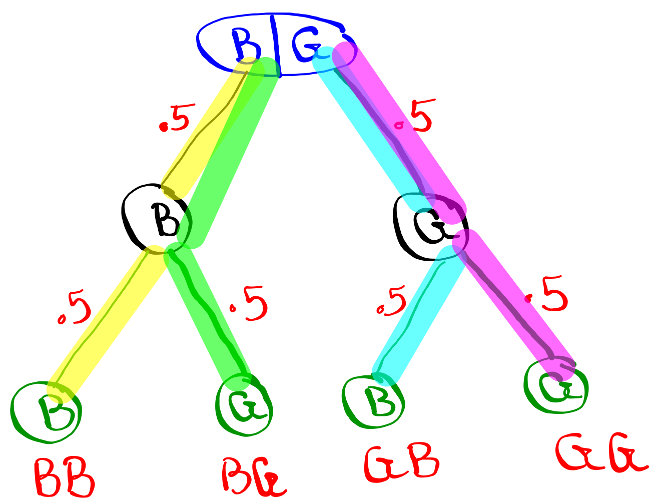
Family: 2 kids

B → Boy

G → Girl

$P(B) = .5$

$P(G) = .5$



$P(BB) = (.5)(.5) = \boxed{.25}$

$P(GB) = (.5)(.5) = \boxed{.25}$

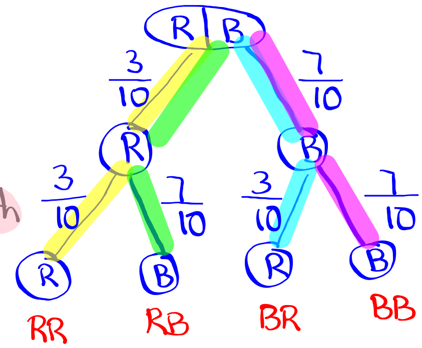
Ex:

A box has 10 Balls.

3 Red

7 Blue

Draw 2 balls with replacement



$P(\text{2 Reds}) = \frac{3}{10} \cdot \frac{3}{10} = \frac{9}{100} = \boxed{.09}$ ✓

$P(1R \,\&\, 1B) = P(RB \text{ or } BR) = \frac{3}{10} \cdot \frac{7}{10} + \frac{7}{10} \cdot \frac{3}{10} = \boxed{.42}$ ✓

$P(\text{2 Blues}) = \frac{7}{10} \cdot \frac{7}{10} = \boxed{.49}$ ✓

# Reds	P(# Reds)
2	.09
1	.42
0	.49

Class QZ Tomorrow 5:50 AM & 8:15 AM	Work on SG 12